NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

TECHNICAL NOTE 2188

EXPERIMENTAL INVESTIGATION OF STIFFENED CIRCULAR

CYLINDERS SUBJECTED TO COMBINED TORSION

AND COMPRESSION

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SUMMARY

Five stiffened circular cylinders were tested to failure under various combinations of torsion and compression. The results of the tests are presented in the form of plots of stringer stress against cylinder load and an interaction curve for the strength of stringers that fail by local crippling. A method is given for calculating stiffener stresses in cylinders subjected to combined loads, and a comparison between calculated and experimental stringer stresses is made.

INTRODUCTION

Little information is available on the load-carrying capacity of skin-stringer structures which are subjected to combined loads. In order to provide information on one phase of this problem, a series of five stiffened circular cylinders were tested to failure under various combinations of torsion and compression in the Langley Structures Research Laboratory. Although the main purpose of the tests was to determine an interaction curve for the strength of stringers that fail by local crippling, data on the stringer stresses were also obtained for stiffened cylinders under combined loads. The test data and a method for estimating stiffener stresses are presented herein.

SYMBOLS

A   cross-sectional area, square inches

E   Young's modulus, kips per square inch

P   applied compressive load, kips

P_{ult} ultimate applied compressive load, kips

T applied torque, inch-kips

T_{ult} ultimate applied torque, inch-kips

R radius of cylinder, inches

R^C_{cr} ratio of compressive load present when panel buckling occurs for cylinder subjected to combined loads to compressive load at panel buckling for cylinder subjected to pure compression

R^T_{cr} ratio of torsional load present when panel buckling occurs for cylinder subjected to combined loads to torsional load at panel buckling for cylinder subjected to pure torsion

R^C_{ult} ratio of compressive load present at failure for cylinder subjected to combined loads to compressive load at failure for cylinder subjected to pure compression

R^T_{ult} ratio of torsional load present at failure for cylinder subjected to combined loads to torsional load at failure for cylinder subjected to pure torsion

d spacing of rings, inches

h spacing of stringers, inches

k nondimensional diagonal-tension factor

n number of stringers in cylinder

t sheet thickness, inches

α angle of diagonal tension, degrees

ϵ normal strain

η proportion of sheet material effective in carrying load in comparison with total sheet material

μ Poisson's ratio

σ normal stress, kips per square inch

σ_{cr} normal stress in cylinder just prior to sheet buckling, kips per square inch

τ shear stress, kips per square inch $\left(\dfrac{T}{2\pi R^2 t}\right)$

τ_{cr} shear stress in sheet just prior to sheet buckling, kips per square inch

Subscripts:

ST   stringer

RG   ring

Superscripts:

C   compression

T   torsion

TEST SPECIMENS AND TEST PROCEDURE

Test Specimens

The test specimens consisted of five circular cylinders of 24S-T3 aluminum alloy. The nominal dimensions of the five cylinders were the same and are given in figure 1. The cylinders were constructed of thin sheet stiffened longitudinally by Z-section stringers and circumferentially by Z-section rings which were partially cutaway at the stringer locations to pass the stringers through the rings. The sheet thickness, stringer area, and ring area of the individual test specimens are given in the following table:

Cylinder	t (in.)	A_{ST} (sq in.)	A_{RG} (sq in.)
1	0.0253	0.0925	0.251
2	.0260	.0916	.254
3	.0248	.0918	.254
4	.0250	.0915	.257
5	.0248	.0915	.257

The sheet thicknesses represent the average of a large number of micrometer measurements. The cross-sectional areas of the stringers and rings were determined by weighing and are believed to be accurate to within 1 percent.

Test Procedure

The cylinders were secured at one end to a rigid support while a torque and a compressive load were applied to the other end through suitable test rigs by hydraulic jacks which were accurate to about 1 percent. The weight of that part of the test rig which was fastened to the cylinders was counterbalanced by weights to eliminate bending in the cylinders. The compressive load was applied to the cylinder through a thrust bearing to allow the cylinder to twist under the action of the torque which was acting simultaneously with the compressive load. A photograph of the test setup is given in figure 2.

Before the beginning of each test, a ratio of torsional load to compressive load was chosen, and an effort was made to keep this ratio the same throughout the test by applying the loads simultaneously in increments of about 5 percent of the estimated ultimate load until failure took place. The loads at which sheet buckling and failure of the cylinder occurred were recorded. In addition, strain measurements on the stringers were recorded at increments of load which were approximately 10 percent of the estimated ultimate load.

The strain measurements on the stringers were made with Baldwin S-R4 type A-1 strain gages. Strain measurements were made on three stringers in the central bay of the cylinders and were converted to stress by multiplying them by Young's modulus which was assumed to be 10.6×10^3 kips per square inch.

TEST RESULTS AND DISCUSSION

Pertinent data obtained from the tests of five stiffened circular cylinders subjected to combined torsional and compressive loads are presented in this section. These data include the loads at which sheet buckling occurred, the stringer stresses at various loads, and the failing loads of the cylinders. A discussion of the data is also given.

Sheet buckling.- The range of load for which buckling of the sheet took place is bracketed by the two dashed lines on the load scale of the stress plots of figure 3. The lower dashed line indicates the load at which the first panel buckled and the top dashed line indicates the lowest load at which all the panels were buckled. Buckling occurred

first, in all cases, in the first unreinforced panels near either end of the cylinder. These end panels buckled at loads from 10 to 25 percent below the loads at which the first panel in the center of the cylinder buckled.

The calculated critical loads are indicated by a dash - short-dash line on the load scale. The critical load for cylinder 1, which was in pure torsion, was calculated with the use of reference 1 and the assumption of simply supported panels. The critical load for cylinder 2, which was in pure compression, was calculated with the use of the curves recommended for design in reference 2. These curves apply to long plates with transverse curvature (with either simply supported or clamped edges) which are subjected to compressive loads. The critical loads for the rest of the cylinders, which were subjected to torsion and compression simultaneously, were computed with the use of the calculated values for cylinders 1 and 2 and a parabolic interaction curve as recommended in reference 2.

The computed critical loads for cylinders 1 and 3 are slightly less than the load at which any panel buckling was observed during the tests of these cylinders, and the computed critical loads for the remaining cylinders lie within the range of load for which panel buckling was observed. (See fig. 3.)

Stringer stresses.- The experimental stringer stresses shown in figure 3 represent the average stress obtained by averaging the stresses from all the gages on the stringers. The average stress is shown rather than individual stress measurements because no significant variations of stress with strain-gage location (either along the stringer or across the cross section of the stringer) were observed from the test data except at loads near failure. For these high loads, individual strain measurements indicated that the stress in the free flange gradually stopped increasing and finally began to decrease with an increase of applied load; whereas, the stress in the web of the stringer increased more and more rapidly with an increase of applied load.

The calculated stringer stresses given in figure 3 were computed with the use of semiempirical formulas developed in the appendix. For cylinder 1 (pure torsion), these formulas reduce to those given in reference 3. For cylinder 2 (pure compression), the formula for the effective-width factor η^C reduces to the Kármán-Sechler formula (reference 4) for the effective width of flat sheet in compression.

The computed stringer stresses for the five cylinders are in reasonable agreement with the stringer stresses as determined by tests. The number of tests is too small, however, to assess the methods of computation used in this paper.

Ultimate strength of test specimens.- If the stringers of a stiffened cylinder fail by column buckling, no interaction problem exists as far as the calculation of the allowable stress is concerned, because the allowable stress does not depend on the manner in which the stringer stress is produced (by end loads on the cylinder or by shear loads producing diagonal tension).

If the stringers fail by local crippling, however, an interaction problem exists; the stringers of the test cylinders were therefore designed to fail by local crippling and did fail in this manner. The loads at which failure occurred are summarized in the following table:

Cylinder	T_{ult} (in.-kips)	P_{ult} (kips)	R^T_{ult}	R^C_{ult}
1	388.0	0	1.00	0
2	0	42.0	0	1.00
3	255.9	26.4	.581	.629
4	129.6	34.6	.334	.825
5	303.0	13.5	.781	.322

The interaction curve of figure 4 was constructed with the use of the data from the preceding table. An analytical expression which is only slightly conservative for the cylinders which were tested under combined loads is, in terms of the usual interaction-curve parameters,

$$\left(R^T_{ult}\right)^{1.5} + R^C_{ult} = 1.00$$

For an interaction curve to be of its greatest usefulness the end points, which correspond to the failing load of the cylinder subjected to pure torsion and pure compression, should be calculable. The end point corresponding to the pure torsion load is discussed in reference 3. This reference gives an allowable stress for stringers which fail by forced crippling. A formula, revised on the basis of more recent tests, is given in reference 5. With the use of the method of reference 3 and the allowable formula for design of reference 5, the failing torque for cylinder 1 was estimated at 374 inch-kips which is about 4 percent conservative. Little information is available concerning the calculation of the end point representing a pure compressive load on the cylinder. Reference 6 treats the problem of local instability of a bare compression member but is not applicable to a compression member which is fastened to a buckled sheet. The latter problem is difficult to treat analytically, and until a satisfactory solution is available,

the only reliable method of obtaining the compression end point for any specific design appears to be to test a panel of dimensions similar to those of the cylinder under consideration.

CONCLUSIONS

A nondimensional interaction curve based on the tests of five cylinders subjected to combined torsion and compression is given. The cylinders failed by local crippling of the stringers. Application of the interaction curve to a particular structure requires that the end points of the interaction curve be known. These end points represent the failing load of a cylinder subjected to pure torsion and pure compression. The end point corresponding to the pure torsion loading can be calculated with the use of existing curved-diagonal-tension theory. No method of comparable reliability appears to be available for computing the end point corresponding to the pure compression loading; to obtain this point, tests may have to be made.

A method for estimating the stiffener stresses in a cylinder under combined loads is developed which gives reasonable agreement with the stresses as determined by tests. Because the method is semiempirical, however, more tests would be desirable in order to assess the accuracy of the method.

Langley Aeronautical Laboratory
    National Advisory Committee for Aeronautics
        Langley Air Force Base, Va., July 6, 1950

APPENDIX

DEVELOPMENT OF FORMULAS FOR STIFFENER STRESSES

Reference 3 gives a set of semiempirical formulas for evaluating the stiffener stresses of torsion cylinders in diagonal tension. These formulas are herein modified to include the case of a combined torsion and compressive load. The modification is such that the formulas of this paper reduce to those of reference 3 if the loading is a pure torque. If the loading is pure compression, the formulas reduce to the Kármán-Sechler formula for effective width of flat sheet in compression (reference 4).

The stringer stress is regarded as consisting of two parts; that is,

$$\sigma_{ST} = \sigma_{ST}^C + \sigma_{ST}^T \qquad (A1)$$

where σ_{ST} is the total stringer stress and σ_{ST}^C and σ_{ST}^T are the parts of the total stringer stress contributed by the compressive load and the torsion load, respectively. The partial stress σ_{ST}^C is assumed to be given by a formula of the usual type

$$\sigma_{ST}^C = -\frac{P}{n\left(A_{ST} + \eta^C ht\right)} \qquad (A2)$$

where the effective-width factor η^C is taken as the Kármán-Sechler value (reference 4) reduced by the factor R_{cr}^C to account for the presence of the torque loading; that is,

$$\eta^C = 0.894\sqrt{\frac{\sigma_{cr}}{\sigma_{ST}^C}}\ R_{cr}^C \qquad (A3)$$

The partial stress σ_{ST}^T is also assumed to be given by an existing formula (from reference 3)

$$\sigma_{ST}^T = -\frac{k\tau \cot \alpha}{\dfrac{A_{ST}}{ht} + \eta^T} \qquad (A4)$$

where the effective-width factor η^T is reduced by R_{cr}^T, or

$$\eta^T = 0.5(1 - k)R_{cr}^T \tag{A5}$$

The interaction curve for panel buckling was used in reducing the effective-width factors (see equations (A3) and (A5)) because the curve is already known from the calculation for the critical stresses σ_{cr} and τ_{cr} and because the choice of interaction curve is not too critical since the effective sheet area is only part of the total compressive area. The ring stress is given by the formula (from reference 3)

$$\sigma_{RG} = - \frac{k\tau \tan \alpha}{\frac{A_{RG}}{dt} + 0.5(1 - k)} \tag{A6}$$

The parameters k and α, appearing in equations (A4) and (A6), are given by the equations:

$$k = \tanh\left[\left(0.5 + 300 \frac{td}{Rh}\right)\log_{10} \frac{\tau}{\tau_{cr}}\right] \tag{A7}$$

and

$$\tan^2\alpha = \frac{\epsilon - \epsilon_{ST}^C - \epsilon_{ST}^T}{\epsilon - \epsilon_{RG} + \frac{1}{24}\left(\frac{h}{R}\right)^2} \tag{A8}$$

where

$$\epsilon = \frac{\tau}{E}\left[\frac{2k}{\sin 2\alpha} + (\sin 2\alpha)(1 - k)(1 + \mu)\right] \tag{A9}$$

and

$$\epsilon^C_{ST} = \frac{\sigma^C_{ST}}{E}$$

$$\epsilon^T_{ST} = \frac{\sigma^T_{ST}}{E} \qquad \Biggr\} \qquad \text{(A10)}$$

$$\epsilon_{RG} = \frac{\sigma_{RG}}{E}$$

The preceding formulas for stiffener stresses must be solved by
successive approximation because of the dependence of the formulas on
each other. Reference 3 gives some charts which facilitate the solution
of equations (A4) to (A10).

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Double sheet thickness in end bays

|←8→|←————45————→|←8→|
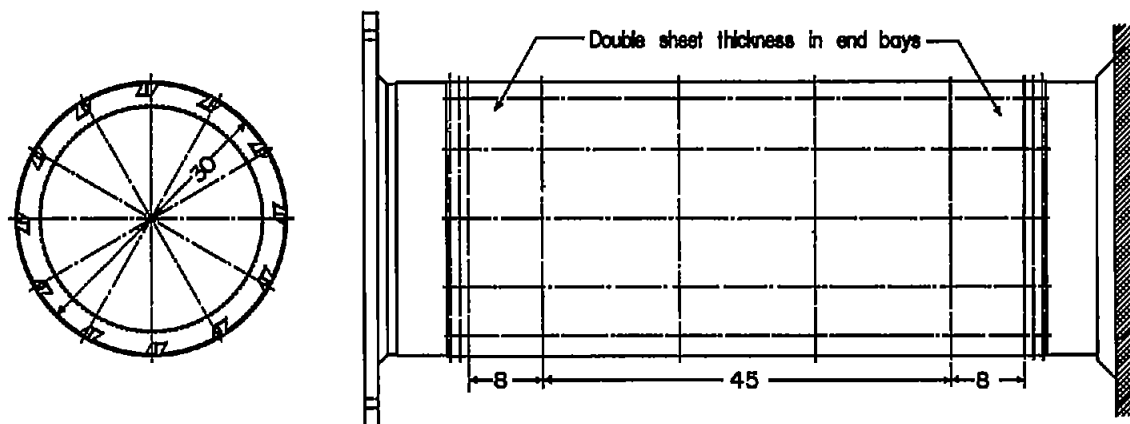
(a) Cylinder dimensions.

0.040→

Stringer

0.08|→

2
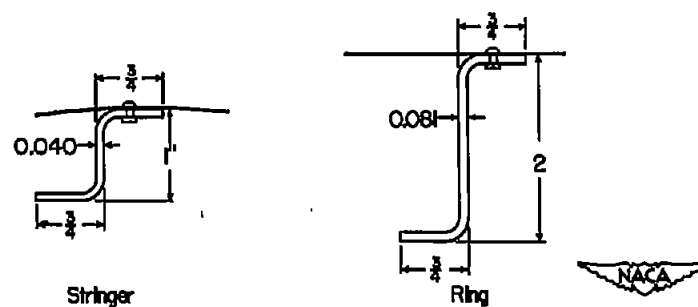
Ring

(b) Stiffener dimensions.

Figure 1.-Nominal dimensions of test cylinders and stiffeners.
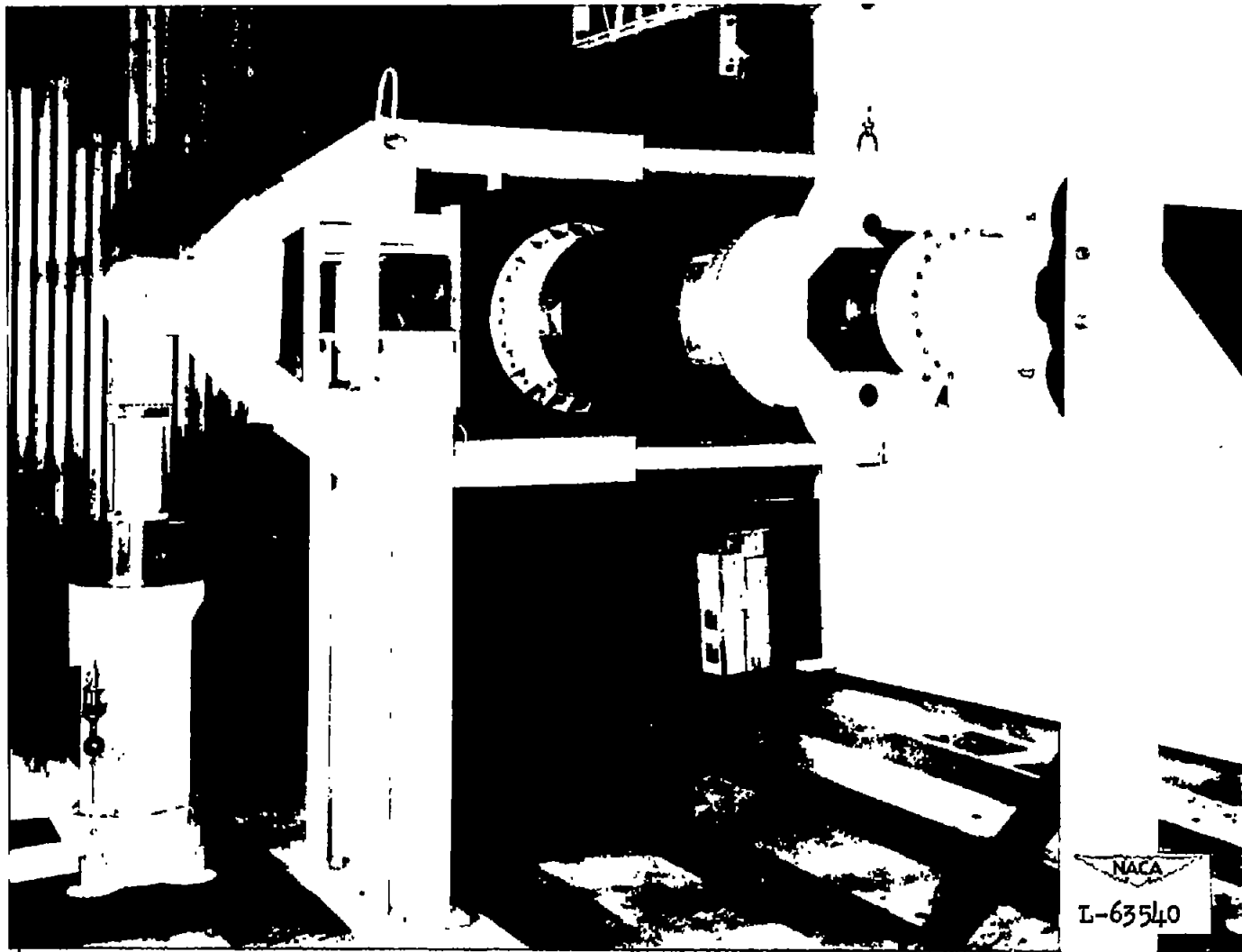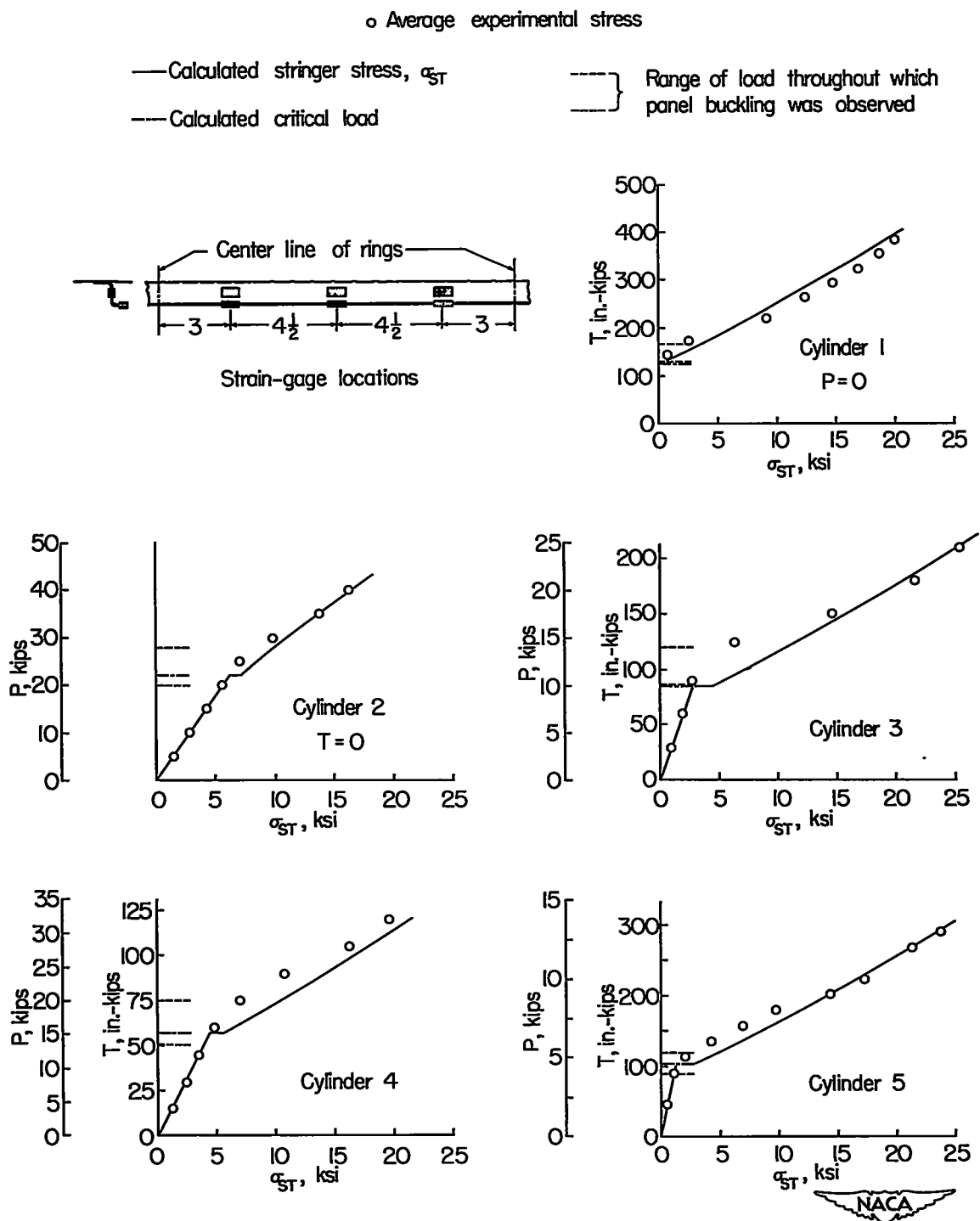
Figure 2.- Test setup.

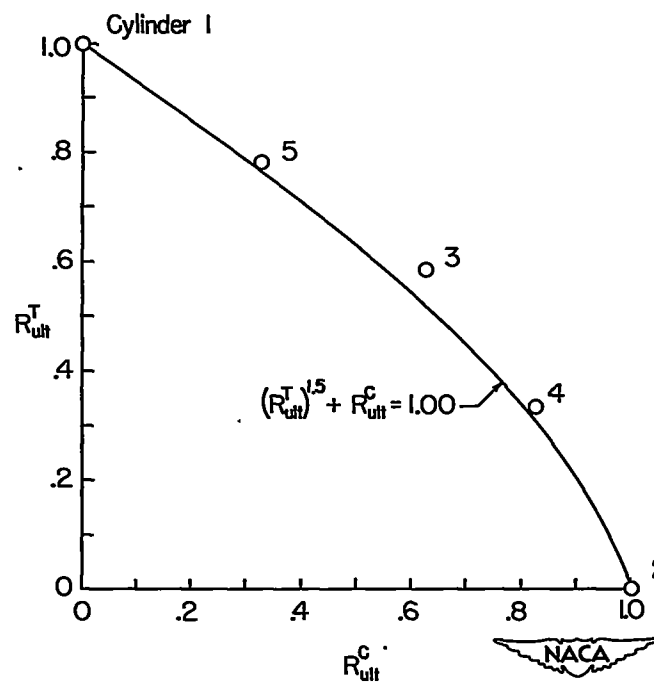Figure 3.-Plots of cylinder load against stringer stress.

Figure 4.- Interaction curve for torsion-compression
    cylinders which failed by local crippling of
    the stringers.